

## Fuzzy identification of T-S model for beam stability control for electron gun\*

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In this paper, Takagi-Sugeno (T-S) fuzzy control is proposed for stabilizing the output beam of accelerators. To model the nonlinear system, we proposed a hybrid optimization algorithm based on quantum-inspired differential evolution and genetic algorithm. Based on the T-S model identified, the corresponding state-feedback fuzzy controller is designed. The method is applied to the LaB<sub>6</sub> electron gun system in the industrial radiation accelerator and the simulation results show its effectiveness.

Keywords: Electron gun system, T-S fuzzy model, Quantum encoding, Model-based control strategy

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### I. INTRODUCTION

Nonlinear control systems based on T-S fuzzy model have attracted lots of attention. It provides a powerful solution for controlling real systems of strong nonlinearity or high degree of uncertainty. This model-based control strategy has been applied to industries of manufacturing, chemicals, aerospace engineering, etc. [1–3].

In this paper, T-S fuzzy control approach is introduced into the beam stability control of high voltage electron accelerators for radiation processing. The electron beam stability is a crucial parameter of accelerators. The factors causing instability of accelerator operation include power source ageing, vacuum variation, voltage stability of the filament, electromagnetic interference, etc. For an E-beam irradiator, the beam stabilization is realized by an electron gun control system that compensates for the beam variations by changing the grid-cathode voltage. The electron gun has the characteristics of nonlinear, time-varying and large inertia, so it is difficult for a traditional PID controller (proportional-integral-derivative controller) to work satisfactorily. The T-S fuzzy control, which controls intelligently with a great adaptive ability, is suitable for such complex nonlinear systems. Using input-output data, we can obtain a fuzzy model of the system represented in the form of a set of fuzzy rules. Each rule is considered as a local linear model, hence the convenience in designing the feedback controller and analyzing stability of the overall closed-loop system. The use of T-S fuzzy control in industrial process control guarantees great accuracy for its universal approximation property, but an inherent drawback remains due to complications in identifying the global fuzzy model, with the proposed control algorithm being computation-intensive and time-consuming. The problem can be solved by selecting controllers with high-performance arithmetic capability and large storage space.

The paper is organized as follows. In Sec. II, we present an optimization algorithm based quantum-inspired differential evolution (QDE) and genetic algorithm (GA). The T-S

fuzzy modeling is checked in Sec. III. Section IV illustrates the detailed design procedure of the T-S fuzzy controller for the LaB<sub>6</sub> electron gun system. Finally, conclusions are summarized in Sec. V.

### II. METHOD OF T-S FUZZY MODELING

The T-S fuzzy model, proposed by Takagi and Sugeno in 1985 [4], is a powerful tool to model and control complex nonlinear systems. For multi-input single-output (MISO) systems, the typical T-S fuzzy model is

$$R_i : \text{if } x_1 \text{ is } A_{i1} \text{ and } x_2 \text{ is } A_{i2} \text{ and } \dots \text{ and } x_m \text{ is } A_{im}, \\ \text{then } y_i(k) = p_{i0} + p_{i1}x_1 + p_{i2}x_2 + \dots + p_{im}x_m \quad (1) \\ (1 \leq i \leq c),$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_m]$  is input variable,  $A_{ij}$  is fuzzy set,  $y_i(k)$  is the output of rule  $R^i$ , and  $P_{ij}$  represents consequent parameter. The model output  $y$  is the weighted average of the individual rule outputs.

$$y = \sum_{i=1}^c w_i y_i \Big/ \sum_{i=1}^c w_i. \quad (2)$$

Given a set of data  $\{x(k), y(k)\} (k = 1, 2, L, N)$ , a discrete T-S fuzzy model can be identified from the input-output data [5]. The whole modeling process involves the following steps:

**Step 1:** Input variables selection. The first task is to select input variables  $x_j (1 \leq j \leq m)$  from process variables. The selection is complex and usually decided according to expert knowledge.

**Step 2:** Fuzzy partition. The input space is divided into  $m * c$  fuzzy sets by fuzzy c-means cluster method.

**Step 3:** Fuzzification. Each input variable  $x_j$  is transformed into fuzzy language with the selected Gaussian membership functions.

$$u_{A_{ij}}(x) = \exp \left( \frac{-(x - ct_{ij})^2}{2\sigma_{ij}^2} \right) \quad (3)$$

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rule 1				.....	rule c			
$ct_{11}$	...	$ct_{1m}$	$\sigma_{11}$	...	$\sigma_{1m}$	.....	$ct_{c1}$	...
$\underbrace{\alpha_1 \dots \alpha_L}_L$				.....			$\underbrace{\alpha_{(2cm-1)L+1} \dots \alpha_{2cmL}}_L$	

Fig. 1. Encoding scheme.

The parameter  $ct_j^i$  is the center of membership function and  $\sigma_j^i$  determines its width.

**Step 4:** Parameter optimization. We propose a hybrid optimization algorithm based QDE and GA to learn the premise parameters  $ct_j^i$  and  $\sigma_j^i$ , and consequent parameters  $p_j^i$ . To start with the proposed algorithm, all parameters are encoded in a quantum bit (Q-bit) chromosome shown in Fig. 1. According to the definition of Q-bit,  $|\alpha_{ij}|^2$  represents the probability that the Q-bit state toward 0 and the value of  $\alpha_{ij}^t$  is chosen in the range of  $[-1, 1]$ .  $L$  is the length of a Q-bit individual. The quantum encoding solution offers a powerful mean to represent the solution space and make the search space larger. Here, the hybrid optimization method is discussed in detail below.

- (1) Generate the initial population. Let  $Q(t) = (q_1^t, q_2^t, \dots, q_M^t)$  denotes the population at generation  $t$ , where  $M$  is the size of the population and  $q_i^t$  is a Q-bit individual defined as  $q_i^t = [\alpha_{i1}^t, \alpha_{i2}^t, \dots, \alpha_{iL}^t]$ .
- (2) Fitness function. The fitness function of each individual is the mean square error ( $MSE$ ) calculated as

$$MSE = \frac{1}{N} \sum_{t=1}^N \left[ \hat{y}(t) - y(t) \right]^2, \quad (4)$$

where  $N$  is the number of input-output data pairs,  $\hat{y}(t)$  is the actual output, and  $y(t)$  is the model output.

- (3) Iteration. The operators of QDE proposed in Ref. [6], which includes mutation, crossover and selection, is employed here to update the Q-bit chromosome. And the three operations are presented in the following.

**Mutation:** The mutant vector  $\mathbf{v}_i^t = [v_{i1}^t, v_{i2}^t, \dots, v_{im}^t]$  on a target vector  $\mathbf{q}_i^t$  is generated by

$$\mathbf{v}_i^t = q_{r1}^t + F(q_{r2}^t - q_{r3}^t), \quad (5)$$

where  $r_1, r_2$  and  $r_3$  are random integers generated from  $\{1, 2, \dots, n\}$  and  $F \in (0, 2)$  is control parameter.

**Crossover:** The crossover operation is to diverse the population. A new vector  $(\mathbf{q}')_i^t = [(q')_{i1}^t, (q')_{i2}^t, \dots, (q')_{im}^t]$  is generated from the target vector  $q_i^t$  and the mutant vector  $v_i^t$ .

$$(\mathbf{q}')_{ij}^t = \begin{cases} v_{ij}^t, & \text{if } rand_{i,j} \leq CR \text{ or } j = J_{rand} \\ \alpha_{ij}^t, & \text{otherwise} \end{cases}, \quad (6)$$

where  $CR \in [0, 1]$  is crossover rate,  $rand_{i,j}$  is a random number that satisfies  $U[0, 1]$  and  $J_{rand}$  is randomly chosen

integer from  $\{1, 2, \dots, m\}$ . To solve the problem of fitness calculation, the observation process defined by the following equation is implemented in each generation.

$$u_{ij}^t = \begin{cases} 1, & \text{if } rand() < 1 - |\alpha_{ij}|^2 \\ 0, & \text{otherwise} \end{cases}, \quad (7)$$

where  $i$  and  $j$  are random integers.

Once the corresponding binary population is observed, the fitness value of the individuals can be evaluated.

**Selection:** For the minimum optimization problem, the individuals with smaller objective function values replace the target vectors and are preserved for the next generation.

$$x_i^{t+1} = \begin{cases} (u')_i^t, & \text{if } f(u_i^t) \leq f(x_i^t) \\ x_i^t, & \text{otherwise} \end{cases}, \quad (8)$$

$$q_i^{t+1} = \begin{cases} (q')_i^t, & \text{if } f(u_i^t) \leq f(x_i^t) \\ q_i^t, & \text{otherwise} \end{cases}, \quad (9)$$

where  $x_i^t$  and  $u_i^t$  are the observed binary individuals on  $q_i^t$  and  $(q')_i^t$ .

GA is also a population-based algorithm like QDE and uses similar operators [7]. The combination is to avoid being trapped in local optimum and to improve efficiency and accuracy of the optimization. Assuming  $P_c \in [0, 1]$  as crossover probability,  $P_m \in [0, 1]$  as mutation probability and  $T$  as the maximum generation, the iteration process is described in the following.

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**Algorithm 1:** Procedure of the hybrid optimization algorithm

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```

begin
  t = 0  initialize Q(t)
  t = t + 1
  while (t ≤ T) do
    make X(t) by observing the state of Q(t)
    generate the mutation operator V(t) using Eq. (5)
    make Q'(t) using the crossover operator in Eq. (6)
    obtain U(t) by observing the state of Q'(t) according to Eq. (7)
    evaluate X(t) and U'(t)
    update X(t + 1) and Q(t + 1) by the selection operator in Eqs. (8) and (9)
    start with the population U'(T) and calculate the fitness of each chromosome in the population
    apply the roulette wheel selection to U'(T)
    perform uniform crossover with probability P_c
    make U'(t + 1) using simple mutation with probability P_m
    replace the current population Q(t) with the new population U'(t + 1)
  end
end

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**Step 5.** The best individual is used to construct the T-S fuzzy model.

TABLE 1. Comparison of different methods for Box-Jenkins data

Method	Inputs	Rules	MSE
Sugeno <i>et al.</i> [8]	3	6	0.190
Li <i>et al.</i> [9]	2	2	0.1608
Our method	3	5	0.1226
Leski [10]	10	2	0.1185

### III. EXPERIMENTAL EXAMPLES

Three kinds of nonlinear system modeling have been conducted to validate capability of the proposed method.

#### A. Identification of the Box-Jenkins gas furnace

The Box-Jenkins gas furnace data set, with 296 pairs of data points  $\{u(k), y(k)\}$ , is considered as a standard experimental data to test the validity of fuzzy modeling, where  $u(k)$  is the input gas rate in  $\text{ft}^3/\text{min}$  and  $y(k)$  is the percentage concentration of output  $\text{CO}_2$ . To construct the fuzzy model,  $\{y(k-1), u(k-3), y(k-2)\}$  is defined as the input variable and  $y(k)$  is the output variable. So the available data are  $k = 4 : 296$ . So, the number of data pairs for identification is  $N = 293$ . The other parameters are  $r = 5$ ,  $M = 40$ ,  $L = 16$ ,  $T = 100$ ,  $F = 0.2$ ,  $CR = 0.6$ ,  $P_c = 0.5$ , and  $P_m = 0.008$ .

The T-S fuzzy model is built and the mean square error of the T-S fuzzy model is 0.1226. Table 1 compares the performance of our method with other modeling technologies. Our model has a smaller error than the other methods except Ref. [10]. Figure 2(a) shows the predicted and actual output values on the gas furnace data. Figure 2(b) shows the prediction error. One can see that the model output fits the experimental data very well.

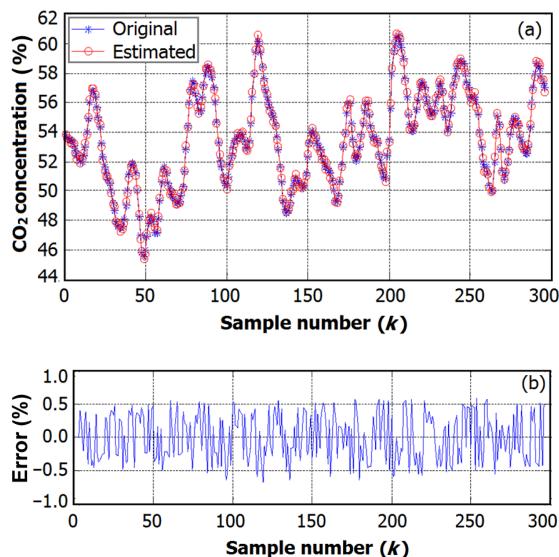


Fig. 2. (Color online) Modeling performance for Box-Jenkins furnace data: (a) the original and estimated data, (b) deviations from the original data.

#### B. Function approximation

We carry out the algorithm on the double input single output (DISO) system described as

$$y = (1 + x_1^{-2} + x_2^{-1.5})^2 \quad 1 \leq x_1, x_2 \leq 5. \quad (10)$$

The goal is to approximate the nonlinear function using a T-S fuzzy model. We obtain 300 data points with a random input signal  $x = (x_1, x_2)$ , where  $x_1$  and  $x_2$  distribute uniformly in Ref. [1, 5]. The  $x_1(k)$  and  $x_2(k)$  are selected as the input variables to predict the output  $y(k)$ . The parameters of the modeling algorithm are  $r = 6$ ,  $M = 40$ ,  $L = 16$ ,  $T = 100$ ,  $F = 0.2$ ,  $CR = 0.6$ ,  $P_c = 0.5$ , and  $P_m = 0.2$ .

Figure 3 shows the modeling performance by comparing the modeled and actual outputs, and giving the prediction errors. The modeling error ( $MSE$ ) is 0.0756 (Table 2), being better than those of the other two methods. So, our method is effective on nonlinear plant approximation problem.

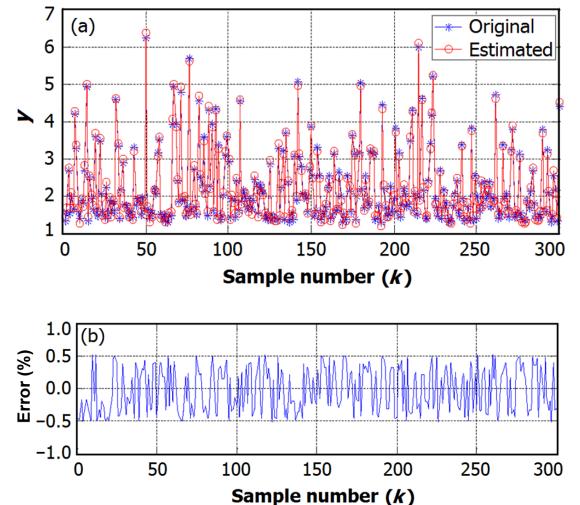


Fig. 3. (Color online) Modeling performance for DISO nonlinear system: (a) the original and estimated data, (b) deviations from the original data.

TABLE 2. Results for function estimation using different methods

Method	Inputs	Rules	MSE
Sugeno <i>et al.</i> [11]	2	6	0.079
Gómez-Skarmeta <i>et al.</i> [12]	2	5	0.09
Our method	2	6	0.0756

#### C. UCI datasets

In this section, the evaluation of the proposed modeling approach is based on two datasets from the UCI repository: auto-mpg dataset and Boston housing dataset. The auto-mpg problem is to predict the fuel consumption (mpg) for automobile based on 4 continuous attributes (displacement, horse-

power, weight, acceleration) and 3 multi-valued discrete attributes (cylinders, model year, origin). The attribute ‘car name’ is omitted here and a total of 392 observations are collected for the prediction. The Boston housing problem is to predict the median value of owner-occupied homes in suburbs of Boston using 12 continuous inputs and one binary-valued input. The database has 506 observations. Due to different types of input variables, a mixed quantum encoding scheme is used. The discrete and continuous variables are encoded respectively.

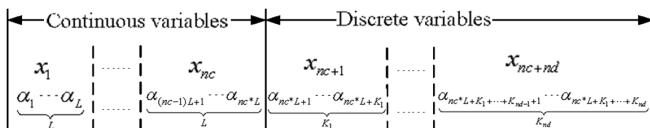


Fig. 4. The mixed coding.

As shown in Fig. 4,  $nc$  and  $nd$  are numbers of continuous variables and discrete variables in each rule.  $K$  is the length of discrete variable, which satisfies  $2^K \geq M$  ( $M$  is the number of possible values of each discrete variable). The parameters of auto-mpg are  $r = 5$ ,  $M = 40$ ,  $L = 16$ ,  $K_1 = 2$ ,  $K_2 = 4$ ,  $K_3 = 2$ ,  $T = 100$ ,  $F = 0.2$ ,  $CR = 0.6$ ,  $P_c = 0.5$ , and  $P_m = 0.2$ . The parameters of Boston housing are set the same as auto-mpg but the length of discrete variable CHAS is  $K = 1$ .

TABLE 3. Comparison of different methods for auto-mpg and Boston housing

Data sets	Method	Inputs	Rules	MSE
Auto-mpg	Castellano <i>et al.</i> [13]	2	4	7.8961
	Gaweda <i>et al.</i> [14]	2	3	7.0756
	Abonyi <i>et al.</i> [15]	2	4	5.6169
	Our method	7	5	0.4502
Boston housing	Han <i>et al.</i> [16]	6	2	8.2944
	Huang <i>et al.</i> [17]	4	16	2.7225
	Our method	13	5	0.6237

Using this modeling method,  $MSE$  for both training datasets are 0.4502 and 0.6237, respectively. The identification error of the proposed algorithm is compared with the performance of some other methods, as shown in Table 3. It can be seen that the proposed algorithm performs better than other algorithms in terms of  $MSE$ . The modeling performance of the fuzzy models and the prediction error are shown in Fig. 5. It can be seen that the prediction error is small and it can approximate the actual output well with fewer rules. The results indicate that the proposed algorithm has a good prediction capability.

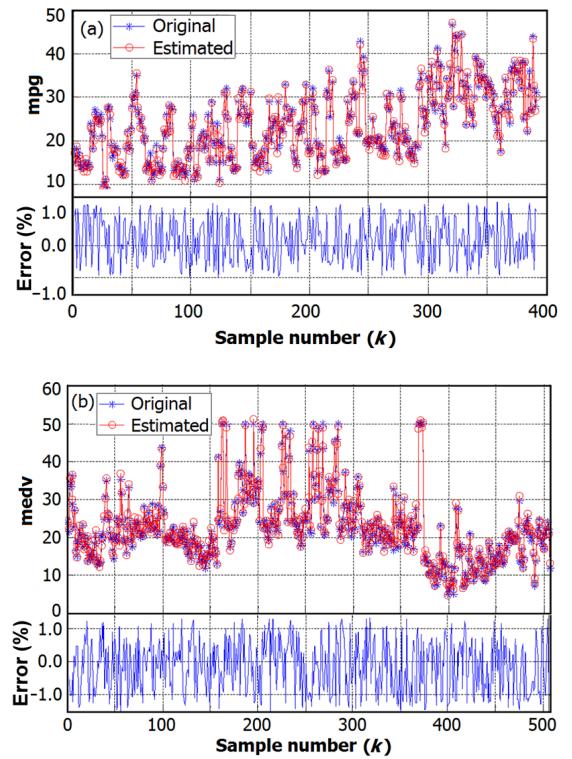


Fig. 5. (Color online) Modeling performance for (a) auto-mpg and (b) Boston housing datasets.

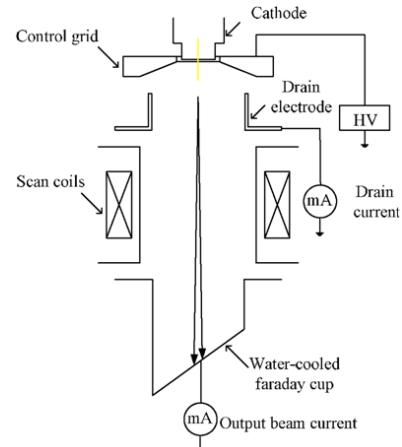


Fig. 6. (Color online) Schematic diagram of the  $\text{LaB}_6$  electron gun system.

#### IV. T-S FUZZY CONTROL FOR $\text{LaB}_6$ ELECTRON GUN SYSTEM

##### A. Model identification

The  $\text{LaB}_6$  electron gun experiment platform (Fig. 6) mainly consists of a  $\text{LaB}_6$  cathode electron-gun, a 40 kV/500 mA stabilized high voltage power supply, a high voltage isolated power supply for the  $\text{LaB}_6$  filament, a water-cooled Faraday

cup, beam current measurement system, beam scanning system and vacuum system. In the experiment, all measurements were carried out in normal operating conditions. Firstly the  $\text{LaB}_6$  cathode was preheated for activating purpose. Then the electron beam was adjusted by changing the filament voltage of the electron gun manually. During the test, 52 input-output data pairs (filament voltage  $V_f$  and output beam current  $I_e$ ) were recorded while operating at a high DC voltage of 27.5 kV.

According to the modeling method above,  $X = [V_f(k-1), I_e(k-1), I_e(k-2), I_e(k-3)]$  was selected as the input variable and  $I_e$  was defined as the output variable. So  $N = 49$  data pairs were available. In order to simplify the structure of the controller, the number of fuzzy rules was set as  $r = 6$ . The other parameters were  $M = 40$ ,  $L = 16$ ,  $T = 100$ ,  $F = 0.1$ ,  $CR = 0.5$ ,  $P_c = 0.5$ , and  $P_m = 0.2$ . After identification from the measured data pairs, the final T-S fuzzy model is described as the following:

$$\begin{aligned}
 R^1 : & \text{if } I_e(k-1) \text{ is } U(18.7397, 2.5292) \text{ and } I_e(k-2) \text{ is } U(15.9169, 4.0700) \text{ and } I_e(k-3) \\
 & \text{is } U(13.7828, 13.0126) \text{ and } V_f(k-1) \text{ is } U(186.5834, 2.5872), \\
 & \text{then } I_e(k) = 0.8085I_e(k-1) + 0.4888I_e(k-2) - 0.6445I_e(k-3) - 0.0079V_f(k-1); \\
 R^2 : & \text{if } I_e(k-1) \text{ is } U(2.8225, 8.2622) \text{ and } I_e(k-2) \text{ is } U(2.6088, 13.2683) \text{ and } I_e(k-3) \\
 & \text{is } U(2.4146, 8.7800) \text{ and } V_f(k-1) \text{ is } U(162.5288, 17.7748), \\
 & \text{then } I_e(k) = -0.3957I_e(k-1) - 0.4018I_e(k-2) + 3.0076I_e(k-3) + 0.0465V_f(k-1), \\
 & u(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}, \quad (x \in U(c, \sigma)). \tag{11}
 \end{aligned}$$

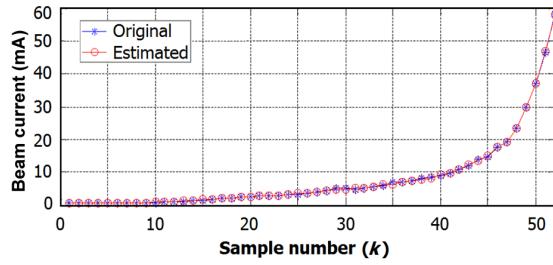


Fig. 7. (Color online) Simulation results of  $\text{LaB}_6$  electron gun data.

The simulation results is shown in Fig. 7. The fuzzy model

approximate the electron gun system very well. The modeling error is just  $MSE = 0.1593$ .

## B. State feedback control based on T-S fuzzy model

Model-based control design is considered to be an important application [18, 19]. For the input-output T-S fuzzy model in Eq. (11), we define the state vector  $x(k) = [I_e(k), I_e(k-1), I_e(k-2)]^T$  and the input variable  $u(k) = V_f(k)$ . The selection of the state vector for T-S state-space fuzzy model is a little different. The model is useful for control only when each rule share the same state vector [20]. Then we have the following T-S state-space model [21, 22]

$$\begin{aligned}
 R^i : & \text{if } I_e(k-1) \text{ is } M_1^i \text{ and } I_e(k-2) \text{ is } M_2^i \text{ and } I_e(k-3) \text{ is } M_3^i \text{ and } V_f(k-1) \text{ is } M_4^i, \\
 & \text{then } \begin{cases} x^i(k+1) = \mathbf{A}_i x(k) + \mathbf{B}_i u(k) \\ y^i(k) = \mathbf{C}_i x(k) \end{cases} \quad (i = 1, 2). \\
 \mathbf{A}_1 = & [0.8085 \ 0.4888 \ -0.6445; 100; 010], \quad \mathbf{B}_1 = [-0.0079 \ 0 \ 0]^T, \quad \mathbf{C}_1 = [1 \ 0 \ 0], \\
 \mathbf{A}_2 = & [-0.3957 \ -0.4018 \ 3.0076; 100; 010], \quad \mathbf{B}_2 = [0.0465 \ 0 \ 0]^T, \quad \mathbf{C}_2 = [1 \ 0 \ 0]. \tag{12}
 \end{aligned}$$

The state feedback controller designed by PDC means [23–25] is

$$\begin{aligned}
 CR^i : & \text{if } I_e(k-1) \text{ is } M_1^i \text{ and } I_e(k-2) \text{ is } M_2^i \\
 & \text{and } I_e(k-3) \text{ is } M_3^i \text{ and } V_f(k-1) \text{ is } M_4^i, \quad (13) \\
 & \text{then } u^i(k) = r(k) - \mathbf{F}_i x(k) \quad (i = 1, 2),
 \end{aligned}$$

where  $r(k)$  is a reference input and  $\mathbf{F}_i$  is the feedback gain.

The stability analysis and design of the fuzzy controller are important issues for fuzzy control systems. In our work, the controller was designed directly to guarantee stability of the overall system [26]. The stability analysis was based on Lyapunov approach, which was proposed by Tanaka and Sugeno. The theorem can be applied to derive the following stability conditions:

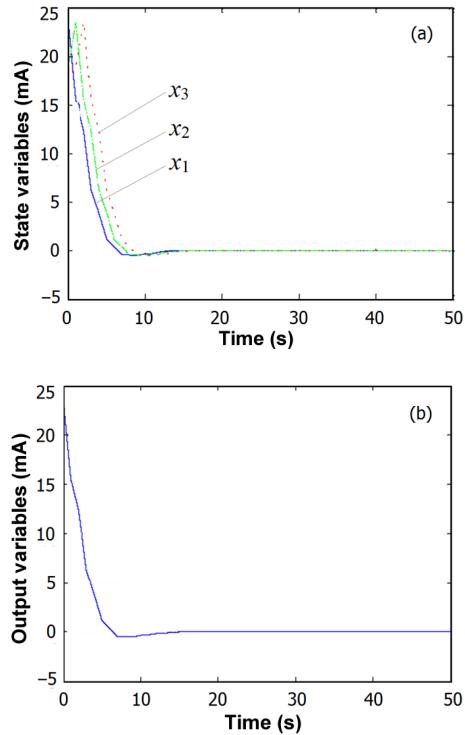


Fig. 8. (Color online) The state-variable response (a) and output-variable response (b), under  $\mathbf{x}_0 = [23.5, 18.17.5]^T$ .

$$\begin{cases} (\mathbf{A}_i - \mathbf{B}_i \mathbf{F}_i)^T \mathbf{P} (\mathbf{A}_i - \mathbf{B}_i \mathbf{F}_i) - \mathbf{P} < 0 \\ \left( \frac{\mathbf{A}_i - \mathbf{B}_i \mathbf{F}_j + \mathbf{A}_i - \mathbf{B}_j \mathbf{F}_i}{2} \right)^T \mathbf{P} \left( \frac{\mathbf{A}_i - \mathbf{B}_i \mathbf{F}_j + \mathbf{A}_i - \mathbf{B}_j \mathbf{F}_i}{2} \right) - \mathbf{P} < 0 \end{cases} \quad (14)$$

Now multiplying both the inequalities on the left and right by  $\mathbf{P}^{-1}$  and defining new variables  $\mathbf{Q} = \mathbf{P}^{-1}$ ,  $\mathbf{N}_i = \mathbf{F}_i \mathbf{Q}$  [27], the inequalities can be rewritten as:

$$\begin{cases} \mathbf{Q} - (\mathbf{A}_i \mathbf{Q} - \mathbf{B}_i \mathbf{N}_i)^T \mathbf{Q}^{-1} (\mathbf{A}_i \mathbf{Q} - \mathbf{B}_i \mathbf{N}_i) > 0 \\ \mathbf{Q} - \left( \frac{(\mathbf{A}_i + \mathbf{A}_j) \mathbf{Q} - \mathbf{B}_i \mathbf{N}_j - \mathbf{B}_j \mathbf{N}_i}{2} \right)^T \times \mathbf{Q}^{-1} \left( \frac{(\mathbf{A}_i + \mathbf{A}_j) \mathbf{Q} - \mathbf{B}_i \mathbf{N}_j - \mathbf{B}_j \mathbf{N}_i}{2} \right) > 0 \end{cases} \quad (15)$$

Based on the Schur complements, the conditions are equivalent to:

$$\begin{cases} \begin{bmatrix} \mathbf{Q} & (\mathbf{A}_i \mathbf{Q} - \mathbf{B}_i \mathbf{N}_i)^T \\ (\mathbf{A}_i \mathbf{Q} - \mathbf{B}_i \mathbf{N}_i) & \mathbf{Q} \end{bmatrix} > 0, \quad i = 1, 2, \dots, r \\ \begin{bmatrix} \mathbf{Q} & \left( \frac{(\mathbf{A}_i + \mathbf{A}_j) \mathbf{Q} - \mathbf{B}_i \mathbf{N}_j - \mathbf{B}_j \mathbf{N}_i}{2} \right)^T \\ \left( \frac{(\mathbf{A}_i + \mathbf{A}_j) \mathbf{Q} - \mathbf{B}_i \mathbf{N}_j - \mathbf{B}_j \mathbf{N}_i}{2} \right) & \mathbf{Q} \end{bmatrix} > 0, \quad i < j \leq r \end{cases} \quad (16)$$

The newly defined variables  $\mathbf{Q}$  and  $\mathbf{N}_i$  can be obtained. Finally, the common positive definite matrix is  $\mathbf{P} = \mathbf{Q}^{-1}$  and the state feedback gains are calculated by  $\mathbf{P}_i = \mathbf{N}_i \mathbf{Q}^{-1}$ .

Considering the system described in Eq. (11), the coefficient matrixes are already known. So we have:

$$\mathbf{P} = \begin{bmatrix} 0.0177 & -0.0009 & -0.0035 \\ -0.0009 & 0.0070 & -0.0005 \\ -0.0035 & -0.0005 & 0.0045 \end{bmatrix}, \quad (17)$$

$$\mathbf{F}_1 = [-17.6663 \ -15.1942 \ 66.4235],$$

$$\mathbf{F}_2 = [-11.5028 \ -13.4181 \ 65.0920].$$

It can be verified that each subsystem is locally stable by calculating its closed-loop poles. And existence of the common positive definite matrix  $\mathbf{P}$  guarantees stability of the overall close-loop system. Simulation was performed to demonstrate effectiveness of the electron gun controller, with the initial condition of  $\mathbf{x}_0 = [23.5, 18.17.5]^T$ , Fig. 8(a) shows the trajectory of  $x_1(k)$ ,  $x_2(k)$  and  $x_3(k)$  under control of the

proposed state-feedback controller, and in Fig. 8(b) the output is globally asymptotically stable.

## V. CONCLUSION

In this paper, we have proposed a novel hybrid optimization algorithm based on QDE and GA. And it is applied successfully to the T-S fuzzy modeling procedure of the LaB<sub>6</sub> electron gun system. The simulation results show that the modeling method has a strong ability to modeling complex nonlinear systems. Furthermore, we design a state-feedback fuzzy controller based on T-S state-space fuzzy model using the PDC method. The design of the fuzzy controller and the stability analysis of the fuzzy control system have been completed at the same time. The approach is considered to be more practical and the simulation results show that the proposed T-S fuzzy control system has good control stability.

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